1901MNRAS..62...41S

(*a*) the ephemeris of the Earth can be computed once for all; the ephemeris of the planet can with comparatively small labour be improved from time to time as we proceed, and the corrections can be carried into our equations of condition with the minimum of trouble. (*b*) The interpolations from the heliocentric ephemerides are not so troublesome as from a geocentric; the quantities vary more regularly. (*c*) The greater part of the calculation is done on a machine instead of with logarithms, which saves a deal of writing. (*d*) The form of the work is comparatively simple, and you see clearly what you are doing all through.

(5) Since the sub-committee of the *Comité International Permanent* is publishing the revised places of the standard stars referred to Newcomb's Fundamental Catalogue, it will be necessary to refer the elements of the orbit of *Eros* to Newcomb's system—at present they are based on Leverrier's—and to compute the ephemeris of the Earth from Newcomb's tables of the Sun, with Gill's value for the mass of the Moon.

(6) Finally, the great advantage is that photographic results reduced on these lines by different observatories would be most readily available for a general combination. And the micrometric results could be easily included, for they are essentially measures in rectangular coordinates, and they might be reduced in exactly the same way.

Cambridge Observatory:
1901 *November* 6.

The Determination of Selenographic Positions and the Measurement of Lunar Photographs.

[Second Paper.]

Determination of a first group of Standard Points by Measures made at the Telescope and on Photographs. By S. A. Saunder, M.A.

§ 1. *Recapitulation of First Paper.*

In a previous paper (*Monthly Notices*, vol. lx. p. 174) I called attention to the unsatisfactory state of our knowledge of the exact positions of the lunar formations, and to the increase in accuracy which might be obtained by measuring from the well-determined point *Mösting A*; formulæ were developed for reducing the measures, and a few results were given.

It was also shown that by the measurement of such photographs as those now being taken at the Paris Observatory a great increase might be made in the number of points whose positions could be accurately determined without necessitating

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1901MNRAS..62...41S

an inordinate amount of computation. Formulæ were developed for reducing the measures on the assumption that the positions of a number of points were known with sufficient accuracy, but it was found that the accepted positions of "points of the first order" were not accurate enough to give the best results obtainable from the photographs, and a second method was developed depending partly on measures of the limb, partly on a combination of measures of the same points on two different photographs, somewhat analogous to the optical combination of two pictures by the stereoscope. Some tentative results were given, and in conclusion I expressed my intention of attacking other pairs of plates in the same way.

Further work on the same pair however showed that the constants obtained ought to be capable of improvement; there were signs of a progressive error, which I now know to have been due to an error of scale on one of the plates. This error was not of sufficient magnitude to vitiate the general conclusions drawn in the paper. I soon became convinced that no really satisfactory solution was likely to be obtained so long as any part of the work depended on measures of the limb. I then tried to determine the plate constants by an extension of what I have called the stereoscopic method, and when this failed, as will be described below, I had recourse to the telescope, and proceeded to determine the positions of a number of points surrounding *Mösting A* at distances not much exceeding 500″, this limit being imposed by the construction of my micrometer.

§ 2. *Telescopic Measures on the Moon.*

These measures were all made and reduced as described in § 3 to § 6 of the previous paper, the only change in detail being that, with the notation of § 4, when ξ', η', ζ' have been once measured the equation $z'=a_3\xi'+b_3\eta'+c_3\zeta'$ has been used to compute the value of $1-z' \sin s'$, the value of z' thus obtained being always sufficiently accurate for the purpose. x', y' have then been determined from the equations at the bottom of p. 177.

$$x'=x\frac{1-z' \sin s'}{1-z \sin s'} -Q(1-z' \sin s') \sin (P-C)$$

$$y'=y\frac{1-z' \sin s'}{1-z \sin s'} +Q(1-z' \sin s') \cos (P-C)$$

This saves the steps required to compute the quantities denoted by x_1', y_1', z_1'.

When the work was nearly completed I found that Dr. Franz of Breslau had already applied the method of measurement from *Mösting A* to determine the positions of eight other points, his results having been published in an Appendix to vol. xxxviii. of the Königsberg Observations a short time before my paper was read (see also *Monthly Notices*, vol. lxi. p. 265).

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1901MNRAS..62...41S

These measures have been of great use to me as will be explained below. The places expressed in terms of rectangular coordinates are :

Proclus ...	$\xi = +\cdot70222 \pm$	$\cdot00012$	$\eta = +\cdot27698 \pm$	$\cdot00020$
Macrobius *a* ...	$+\cdot61034 \pm$	12	$+\cdot33455 \pm$	20
Sharp A ...	$-\cdot45663 \pm$	23	$+\cdot73763 \pm$	15
Aristarchus ...	$-\cdot67551 \pm$	17	$+\cdot40201 \pm$	21
Gassendi *z* ...	$-\cdot65246 \pm$	12	$-\cdot28330 \pm$	15
Byrgius A ...	$-\cdot81612 \pm$	17	$-\cdot41561 \pm$	20
Nicolai A ...	$+\cdot29598 \pm$	11	$-\cdot67494 \pm$	14
Fabricius K ...	$+\cdot46642 \pm$	16	$-\cdot72018 \pm$	17

The measures were all made with the Königsberg heliometer, and the observations extend over a period of six years. *Byrgius A* was measured on eight different nights ; the others depend each on from eleven to thirteen separate determinations.

An account of the measures of *Ptolemæus A* and *Triesnecker B*, kindly made for me by Professor Barnard, has already appeared (*Monthly Notices*, vol. lx. p. 540), together with the particulars of my own measures of these points (see also *Astronomical Journal*, vol. xxii. p. 33).

Four measures of *Dollond* are given in *Monthly Notices*, vol. lx. p. 182.

These, with *Mösting A* and those given below, complete the list of the positions used in the reduction of the photographic plates.

A complete set of measures consists, as described in § 6 of my previous paper, of six settings for position-angle, five double distances, and six more settings for position-angle. Occasionally a set has been interrupted by clouds or by failure of definition. If less than half the whole set had been made, no account has been taken of them ; if more than half, they have been reduced and allowed weight $\frac{1}{2}$ in the final means. No place depends on less than four complete sets or their equivalent.

The probable error of a coordinate derived from one complete set, as determined from the whole of the residuals, is ·00022, and hence the probable error of the mean of four such measures is ·00011, the geocentric value of which is 0″·10.

Results of micrometric measures made on the Moon with a 7-inch O.G.

Hesiodus B.

			Residual.		Residual.	
1899 Dec.	14	$\xi = -\cdot2670 +$	$\cdot0001 \times \frac{1}{2}$	$\eta = -\cdot4553 +$	$\cdot0002 \times \frac{1}{2}$	Wt. $\frac{1}{2}$
	15	$-\cdot2673 -$	2	$-\cdot4554 +$	1	
1900 Feb.	11	$-\cdot2673 -$	2	$-\cdot4558 -$	3	
Nov.	9	$-\cdot2664 +$	7	$-\cdot4554 +$	1	
1901 Apr.	1	$-\cdot2676 -$	5	$-\cdot4555$	0	
		$-\cdot2671$		$-\cdot4555$		

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1901MNRAS..62...41S

Nicollet.

			Residual.		Residual.
1900 Aug.	17	$\xi = -\cdot1995+$	$\cdot0003$	$\eta = -\cdot3729+$	$\cdot0001$
Oct.	13	$-\cdot1999-$	1	$-\cdot3732-$	2
1901 Jan.	29	$-\cdot1997+$	1	$-\cdot3732-$	2
Feb.	1	$-\cdot2001-$	3	$-\cdot3726+$	4
		$-\cdot1998$		$-\cdot3730$	

Thebit A.

1900 Apr.	10	$\xi = -\cdot0785+$	$\cdot0007$	$\eta = -\cdot3672+$	$\cdot0002$
	13	$-\cdot0795-$	3	$-\cdot3675-$	1
May	7	$-\cdot0792$	0	$-\cdot3671+$	3
Nov.	10	$-\cdot0796-$	4	$-\cdot3676-$	2
		$-\cdot0792$		$-\cdot3674$	

Abenezra b.

1900 Sept.	12	$\xi = +\cdot1634-$	$\cdot0001$	$\eta = -\cdot3555-$	$\cdot0003$
Oct.	10	$+\cdot1635$	0	$-\cdot3546+$	6
1901 Jan.	31	$+\cdot1633-$	2	$-\cdot3557-$	5
Feb.	28	$+\cdot1636+$	1	$-\cdot3551+$	1
		$+\cdot1635$		$-\cdot3552$	

Airy A.

1900 Sept.	12	$\xi = +\cdot1276$	$\cdot0000$	$\eta = -\cdot2927+$	$\cdot0002$
Oct.	10	$+\cdot1275-$	1	$-\cdot2929$	0
1901 Jan.	29	$+\cdot1281+$	5	$-\cdot2927+$	2
Mar.	3	$+\cdot1274-$	2	$-\cdot2931-$	2
		$+\cdot1276$		$-\cdot2929$	

Lubiniezky B.

1900 Apr.	13	$\xi = -\cdot3863$	$\cdot0000$	$\eta = -\cdot2512+$	$\cdot0003$
Aug.	17	$-\cdot3865-$	2	$-\cdot2515$	0
Oct.	14	$-\cdot3865-$	2	$-\cdot2514+$	1
1901 Jan.	2	$-\cdot3861+$	2	$-\cdot2517-$	2
		$-\cdot3863$		$-\cdot2515$	

Hipparchus C.

1900 Apr.	13	$\xi = +\cdot1420-$	$\cdot0003$	$\eta = -\cdot1288-$	$\cdot0003$
May	7	$+\cdot1424+$	1	$-\cdot1290-$	5
Aug.	13	$+\cdot1423$	0	$-\cdot1283+$	2
1901 Jan.	2	$+\cdot1423$	0	$-\cdot1281+$	4
		$+\cdot1423$		$-\cdot1285$	

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1901MNRAS..62...41S

Euclides.

			Residual.		Residual.
1899	Dec. 13	$\xi = -·4886$	·0000	$\eta = -·1279 +$	·0004
	15	$-·4887-$	1	$-·1283$	0
1901	Feb. 11	$-·4883+$	3	$-·1283$	0
	Nov. 10	$-·4888-$	2	$-·1286-$	3
		$-·4886$		$-·1283$	

Hipparchus G.

1900	Nov. 9	$\xi = +·1289 +$	·0002	$\eta = -·0871 +$	·0004
	10	$+·1290+$	3	$-·0871+$	4
1901	Jan. 4	$+·1285-$	2	$-·0880-$	5
	Feb. 1	$+·1284-$	3	$-·0878-$	3
		$+·1287$		$-·0875$	

Gambart A.

1900	Mar. 12	$\xi = -·3205 +$	·0005	$\eta = +·0173 +$	·0003
	13	$-·3211-$	1	$+·0170+$	0
	Apr. 10	$-·3210$	0	$+·0171+$	1
	Nov. 9	$-·3214-$	4	$+·0167-$	3
		$-·3210$		$+·0170$	

Cayley.

1900	Aug. 12	$\xi = +·2599 -$	·0004	$\eta = +·0691 +$	·0001
	13	$+·2603$	0	$+·0691+$	1
1901	Jan. 30	$+·2604+$	1	$+·0689-$	1
	Feb. 28	$+·2606+$	3	$+·0690$	0
		$+·2603$		$+·0690$	

Hortensius.

1900	Aug. 17	$\xi = -·4662$	·0000	$\eta = +·1127 -$	·0001
	Oct. 14	$-·4662$	0	$+·1128$	0
1901	Jan. 2	$-·4657+$	5	$+·1134+$	6
	30	$-·4664-$	2	$+·1122-$	6
	31	$-·4666-$	$4 \times \frac{1}{2}$	$+·1127-$	$1 \times \frac{1}{2}$ Wt. $\frac{1}{2}$
		$-·4662$		$+·1128$	

Sosigenes a.

1900	Nov. 10	$\xi = +·3141 -$	$·0004 \times \frac{1}{2}$	$\eta = +·1357 +$	$·0008 \times \frac{1}{2}$ Wt. $\frac{1}{2}$
	Dec. 29	$+·3149+$	$4 \times \frac{1}{2}$	$+·1353+$	$4 \times \frac{1}{2}$ Wt. $\frac{1}{2}$
1901	Jan. 29	$+·3147+$	2	$+·1348-$	1
	Feb. 28	$+·3145$	0	$+·1346-$	3
	Mar. 3	$+·3143-$	2	$+·1346-$	3
		$+·3145$		$+·1349$	

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1901MNRAS..62...41S

Bode B.

			Residual.		Residual.
1900	Sept. 12	$\xi = -\cdot 0526 +$	$\cdot 0003$	$\eta = +\cdot 1522 -$	$\cdot 0003$
	Oct. 13	$-\cdot 0528 +$	1	$+\cdot 1524 -$	1
1901	Jan. 31	$-\cdot 0530 -$	1	$+\cdot 1525$	0
	Feb. 1	$-\cdot 0534 -$	5	$+\cdot 1530 +$	5
		$-\cdot 0529$		$+\cdot 1525$	

Bode A.

1899	Dec. 15	$\xi = -\cdot 0203 -$	$\cdot 0004$	$\eta = +\cdot 1570 +$	$\cdot 0001$
1900	Mar. 9	$-\cdot 0194 +$	5	$+\cdot 1569$	0
	12	$-\cdot 0198 +$	1	$+\cdot 1568 -$	1
	13	$-\cdot 0200 -$	1	$+\cdot 1569$	0
		$-\cdot 0199$		$+\cdot 1569$	

Bessel.

1900	Aug. 12	$\xi = +\cdot 2853 -$	$\cdot 0003$	$\eta = +\cdot 3707 +$	$\cdot 0003$
	13	$+\cdot 2852 -$	4	$+\cdot 3709 +$	5
1901	Jan. 2	$+\cdot 2855 -$	1	$+\cdot 3697 -$	7
	4	$+\cdot 2864 +$	8	$+\cdot 3700 -$	4
	Apr. 1	$+\cdot 2856$	0	$+\cdot 3706 +$	2
		$+\cdot 2856$		$+\cdot 3704$	

§ 3. *Attempt to determine Absolute Places from a comparison of two Photographs only.*

The unsuccessful attempt to improve the constants for the plates denoted by the numbers II., III. in my previous paper, to which I referred in my first paragraph, was made as follows :—

The standard coordinates of a point ξ, η are connected with the plate coordinates x, y and the third coordinate $z = \sqrt{1 - x^2 - y^2}$, as shown in *Monthly Notices*, vol. lx. p. 187, by the equations

$$\xi = Ax + By + Cz$$
$$\eta = Dx + Ey + Fz$$

where A, B . . . are given as functions of the librations and of θ, the orientation of the réseau on the plate. All the coordinates are expressed as fractions of the Moon's radius, and referred to the centre of the Moon as origin.

If a small error $\delta\theta$ has been made in the assumed orientation it was shown (*Monthly Notices*, vol. lx. p. 189) that the corresponding errors in ξ, η are $(Bx - Ay)\,\delta\theta$ and $(Dx - Ey)\,\delta\theta$ respectively.

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1901MNRAS..62...41S

But errors may also have been made in α' β', the assumed réseau coordinates of the centre of the Moon's disc, and in r', the assumed value of the radius, also in réseau units. Let $\delta\alpha'$, $\delta\beta'$, $\delta r'$ be the amounts of these errors in réseau units; $\delta\alpha$, $\delta\beta$, δr the same quantities expressed in terms of the Moon's radius.

Let x', y' be the réseau coordinates of the point as actually read, *i.e.* referred to one corner of the réseau as origin.

Then
$$x=\frac{x'-\alpha'}{r'}, \quad y=\frac{y'-\beta'}{r'}$$

$$\therefore\ \delta x=\frac{-r'\delta\alpha'-(x'-\alpha')\delta r'}{r'^2}$$
$$=-\frac{\delta\alpha'}{r'}-\frac{x'-\alpha'}{r'}\,\frac{\delta r'}{r'}$$
$$=-\delta\alpha-x\delta r$$

Similarly
$$\delta y=-\delta\beta-y\delta r$$

And since
$$z=\sqrt{1-x^2-y^2}$$
$$\therefore\ \delta z=-\frac{x\delta x+y\delta y}{z}$$
$$=\frac{x}{z}\,\delta\alpha+\frac{y}{z}\,\delta\beta+\frac{1-z^2}{z}\,\delta r$$

And hence
$$\delta\xi=\mathrm{A}\delta x+\mathrm{B}\delta y+\mathrm{C}\delta z+(\mathrm{B}x-\mathrm{A}y)\delta\theta$$
$$=\left(\mathrm{C}\,\frac{x}{z}-\mathrm{A}\right)\delta\alpha+\left(\mathrm{C}\,\frac{y}{z}-\mathrm{B}\right)\delta\beta+\frac{\mathrm{C}}{z}\delta r-(\mathrm{A}x+\mathrm{B}y+\mathrm{C}z)\delta r$$
$$+(\mathrm{B}x-\mathrm{A}y)\delta\theta$$
$$=\left(\mathrm{C}\frac{x}{z}-\mathrm{A}\right)\delta\alpha+\left(\mathrm{C}\,\frac{y}{z}-\mathrm{B}\right)\delta\beta+\left(\frac{\mathrm{C}}{z}-\xi\right)\delta r+(\mathrm{B}x-\mathrm{A}y)\delta\theta$$

Similarly
$$\delta\eta=\left(\mathrm{F}\,\frac{x}{z}-\mathrm{D}\right)\delta\alpha+\left(\mathrm{F}\frac{y}{z}-\mathrm{E}\right)\delta\beta+\left(\frac{\mathrm{F}}{z}-\eta\right)\delta r+(\mathrm{D}x-\mathrm{E}y)\delta\theta$$

To determine these corrections thirty-seven points were measured on each of the two plates, the measures were reduced with the approximate values of α', β', r', and θ already obtained, and then thirty-seven equations were made of the type

$$\xi_2+\delta\,\xi_2=\xi_3+\delta\,\xi_3$$

and thirty-seven of the type

$$\eta_2+\delta\eta_2=\eta_3+\delta\eta_3$$

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1901MNRAS..62...41S

where the suffixes indicate the plate on which the measures were made.

These gave seventy-four equations for determining the eight quantities, $\delta\alpha_2$, $\delta\beta_2$, δr_2, $\delta\vartheta_2$, $\delta\alpha_3$, $\delta\beta_3$, δr_3, $\delta\theta_3$.

When these were combined by least squares, and the normal equations solved by Gauss' method of elimination, it was found that so long as one of each of the quantities $\delta\alpha$, $\delta\beta$, $\delta\theta$, δr remained in the equations the coefficients of the first term in each equation ranged between 7·8 and 36·9, but that so soon as all the quantities belonging to one plate had been eliminated the first coefficient of the next equation fell to between ·01 and ·04, according to the order of elimination, showing that the values which would be given by the solution would be of very low weight, whilst on pushing to a numerical result a position of the centre was obtained which was known to be in error by something like three seconds of arc.

The reason for this may be seen by considering any pair of the normal equations; if, for example, we take those in $\delta\alpha_2$, $\delta\alpha_3$, these, as written in the usual form, were

$$37{\cdot}0643\delta\alpha_2-36{\cdot}1251\delta\alpha_3+{\cdot}1831\delta\beta_2+9{\cdot}3175\delta\beta_3+1{\cdot}5436\delta\vartheta_2 \\ +1{\cdot}8799\delta\theta_3-2{\cdot}7882\delta r_2-5{\cdot}1871\delta r_3+{\cdot}0040061=0$$

$$-36{\cdot}1251\delta\alpha_2+37{\cdot}7490\delta\alpha_3-9{\cdot}8515\delta\beta_2+{\cdot}2599\delta\beta_3-1{\cdot}1806\delta\theta_2 \\ -{\cdot}5903\delta\theta_3+2{\cdot}9563\delta r_2+6{\cdot}0044\delta r_3-{\cdot}0037328=0$$

If we put $\delta\alpha_2+\delta\alpha_3=2\mathrm{A}$, $\delta\alpha_2-\delta\alpha_3=2a$, the first two terms become

$${\cdot}9392\mathrm{A}+73{\cdot}1894a \text{ in the first equation,}$$
$$1{\cdot}6239\mathrm{A}-73{\cdot}8741a \text{ in the second equation,}$$

whence we see that while the differences are well determined the sums have very small coefficients; and the same holds true for all the pairs of normal equations.

The interpretation is that the unavoidable errors of observation were such that the difference between the librations was not sufficient to give more than a rough approximation to the absolute values of the required constants.

§ 4. *Formation of Equations by Combination of the Telescope and Photographic Measures.*

The work, however, was not entirely lost, for by stopping the elimination when only half effected it was possible (as explained in *Chauvenet*, vol. ii., Appendix, § 52) to obtain four good equations:

$$\begin{aligned}
\delta\alpha_2&=\mathrm{P}\delta\alpha_3+\mathrm{Q}\,\delta\beta_3+\mathrm{R}\,\delta r_3+\mathrm{S}\,\delta\theta_3+\mathrm{T}\ \pm{\cdot}000031\\
\delta\beta_2&=\mathrm{P}'\delta\alpha_3+\mathrm{Q}'\,\delta\beta_3+\mathrm{R}'\,\delta r_3+\mathrm{S}'\,\delta\theta_3+\mathrm{T}'\ \pm{\cdot}000031\\
\delta r_2&=\mathrm{P}''\delta\alpha_3+\mathrm{Q}''\,\delta\beta_3+\mathrm{R}''\,\delta r_3+\mathrm{S}''\,\delta\theta_3+\mathrm{T}''\ \pm{\cdot}000067\\
\delta\theta_2&=\mathrm{P}'''\delta\alpha_3+\mathrm{Q}'''\delta\beta_3+\mathrm{R}'''\delta r_3+\mathrm{S}'''\delta\theta_3+\mathrm{T}'''\pm{\cdot}000067
\end{aligned}$$

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1901MNRAS..62...41S

where P, Q, R . . . were obtained numerically, and the probable errors are expressed in terms of the Moon's radius, the geocentric value of the largest being 0″·06.

These equations were used to transform the linear equations

$$\delta\xi_2 = \mathrm{F}(\delta\alpha_2, \delta\beta_2, \delta\theta_2, \delta r_2) \qquad \delta\eta_2 = f(\delta\alpha_2, \delta\beta_2, \delta\vartheta_2, \delta r_2)$$

found in § 3 into equivalent linear equations

$$\delta\xi_2 = \mathrm{F}_1(\delta\alpha_3, \delta\beta_3, \delta\vartheta_3, \delta r_3) \qquad \delta\eta_2 = f_1(\delta\alpha_3, \delta\beta_3, \delta\vartheta_3, \delta r_3)$$

in which the coefficients are functions of the position of the point ξ, η, as well as of the constants P, Q, R . . .

With these equations measures made upon Plate II. could be used to determine the corrections to the constants of Plate III., and as Plate II. was taken a little after first quarter, Plate III. a little before third quarter, the two together show the whole disc, with a slight overlap near the central meridian.

But in order to find these corrections it was necessary to know the positions of a number of points on the Moon's surface, and it had already been shown (*Monthly Notices*, vol. lx. p. 184) that those previously found as "points of the first order" were not good enough for the purpose. It was at this stage that use was made of the places found at the telescope.

The available material consisted of the positions of *Mösting A*, from which all the others had been measured, 19 points within about 500″ of *Mösting A* measured by myself, two of which had been also measured by Professor Barnard, and the 8 points, all near the limb, measured by Dr. Franz. Two of these 8, *Aristarchus* and *Byrgius A*, were, in consequence of the brightness of the surrounding regions, too poorly defined to be measurable on the photographs. I had therefore a total of 26 points, of which 8 appeared on Plate II. only, 8 on Plate III. only, and 10 were shown on both plates. These were all the centres of regular circular formations. I had also measured the cones of *Herschel* and *Manilius*, but in consequence of the difficulty in locating the summit of a mountain I decided not to employ any points of this nature.

The selected points having been carefully measured on the photographs, the corresponding values of ξ, η were computed for each, as well as the coefficients of the different terms in the expressions for $\delta\xi$, $\delta\eta$, the approximate values of the plate constants being used for the purpose.

Equating the values of $\xi+\delta\xi$, $\eta+\delta\eta$ found from the photographs to the corresponding values of ξ, η found at the telescope, there were 72 equations for the four quantities $\delta\alpha_3$, $\delta\beta_3$, δr_3, $\delta\theta_3$. But these were not all dealt with in one group.

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1901MNRAS..62...41S

§ 5. *Comparison of Constants obtained from Dr. Franz's Measures with those obtained from my own.*

The first point about which I was anxious to satisfy myself was as to the accordance between the positions which I had found and those measured by Dr. Franz. I had already found (*Monthly Notices*, vol. lx. p. 540) a satisfactory agreement between my own measures and measures of the same points by Professor Barnard, but in this case the distances measured had been under 160″. My own micrometer screw had begun to show signs of wear, evidenced by a want of constancy of the error of runs in different parts of the scale, and I felt anxious as to any possible effect of this upon the measures.

I therefore picked out the 60 conditional equations derived from the positions of *Mösting A* and of the points which I had measured myself, and from these determined the corrections to the plate constants.

I then dealt in the same way with the 16 equations obtained from the positions of *Mösting A* and of the points measured by Dr. Franz.

The following table shows the results of the comparison. The values of $\delta\alpha_3$, $\delta\beta_3$, δr_3 are expressed first in terms of the Moon's radius as unit, and secondly in seconds of arc at mean distance of the Moon ; $\delta\theta_3$ is expressed first in circular measure, and secondly in seconds of arc.

	$\delta\alpha_3$	$\delta\beta_3$	δr_3	$\delta\theta_3$
Difference between absolute values of corrections found from the two groups of equations ...	·00023 = 0″·21	·00022 = 0″·21	·00030 = 0″·28	·00002 = 4″
P.E. from measures by S. A. S ...	·00005 = 0·05	·00005 = 0·05	·00015 = 0·14	·00015 = 31
P.E. from measures by J. F. ...	·00011 = 0·10	·00010 = 0·09	·00015 = 0·14	·00014 = 29

The first point to which attention was directed was that the differences between the values of $\delta\alpha_3$, $\delta\beta_3$ were greater than the sum of the probable errors, whilst the probable errors from my own points were about half those from Dr. Franz's points. The fact that my values depended on nearly four times as many conditional equations would explain the difference in probable error, but the difference in the values found I attributed chiefly to the fact that the points which I had measured at the telescope were more clearly defined on the photographs than those which Dr. Franz had measured, so that the photographic measures of my points were more trustworthy than those of Dr. Franz's. It did not appear likely that the position of the centre as determined from my points alone would be subject to any systematic error, and, moreover, when the whole of the 72 conditional equations were solved in one group the probable errors of $\delta\alpha_3$, $\delta\beta_3$ were ·000059 and ·000058 respectively, a little greater than those determined from my points alone. It therefore appeared

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1901MNRAS..62...41S

that the centre would be best determined by considering only the equations of the first group.

The want of definition of Dr. Franz's points upon the photographs would also affect the values of the radius and orientation derived from them; but this was compensated by their greater distance from the centre, and the probable errors from the two groups were practically the same. But in the normal equations for δr_3 the coefficient of δa_3 was nearly as great as that of δr_3. This was found to be due, as to *Mösting A* and the points which I had measured, to a relation between the libration and the position of the terminator in consequence of which the coefficients of δa_3, δr_3 retained the same sign in all the equations of the ξ group given by Plate II. As to the points measured by Dr. Franz the fact that four of them were on one side of the central meridian and only two of them on the other operated in the same way. The result was that the value found for δr_3 depended largely upon that assigned to δa_3, whilst the normal equation for δa_3 was such that the value of δa_3 was only slightly affected by that of δr_3. When this was noticed the normal equations in δr_3, $\delta \theta_3$ given by my points and those given by Dr. Franz's points were solved with the same values of δa_3, $\delta \beta_3$. The difference between the two values found for δr_3 was then reduced to ·00015 of the Moon's radius$=0''\cdot14$. The difference in the scales derived from the two sets of measures amounted therefore to but little more than one part in 7,000, with which agreement I was quite satisfied, and from which it would appear that the wear of the screw had not been sufficient to introduce sensible error into the results.

The two values of $\delta \theta_3$ were practically coincident, though the probable errors show that this was partly fortuitous.

§ 6. *Final Solution of the Equations.*

Having decided that the whole 72 equations should be employed in the determination of δr_3, $\delta \theta_3$, whilst the values of δa_3, $\delta \beta_3$ should depend upon the group of 60 only, the proper course would have been to express δr_3, $\delta \theta_3$ as functions of δa_3, $\delta \beta_3$ from the 72 equations, to substitute these values in the group of 60, and to solve the resulting equations for δa_3, $\delta \beta_3$. This, however, would have involved a considerable addition to the labour of computation, which was already very great, whilst the effect upon the solution would have been almost imperceptible. The greatest difference between the solutions obtained entirely from one set or entirely from the other set of equations amounted only to $0''\cdot2$. I therefore compromised matters by taking the two normal equations in δa_3, $\delta \beta_3$ from the group of 60 conditional equations, the two in δr_3, $\delta \theta_3$ from the whole group of 72, and accepted as final the values resulting from their simultaneous solution.

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1901MNRAS..62...41S

Having determined these the corrections $\delta\alpha_2$, $\delta\beta_2$, δr_2, $\delta\theta_2$ were given by the equations given at the beginning of § 4, and a set of constants was determined which would enable the standard coordinates to be found for any point measurable on either plate.

It should be noted that the coordinates of the 10 points measured at the telescope and shown on both plates enter each into two conditional equations, being equated separately to the corresponding coordinate from each photograph; but those of the 16 points shown on one plate only enter each into only one conditional equation, and so have less weight in the normal equations than the others. But I believe that the principal errors at present are in the photographic measures, and that consequently the course adopted is justifiable.

The advantages which would seem to be secured by expressing the constants of one plate in terms of those of the other are, first that a greater number of conditional equations is obtained than could be secured from either plate alone, and secondly that the use of two plates taken under different librations and oppo-

Comparison between Places derived from Paris Plates II. and III., Professor Barnard and

	Values of ξ.					
	Plate II.	Plate III.	II.–III.	Mean of II. & III.	Telescopic.	Photographic – Telescopic.
Lilius (sm. crater)*	+·0617	+·0620	–·0003	+·0618	...	...
Licetus (sm. crater)†	+·0231	+·0221	+ 10	+·0226	...	...
Fabricius K	+·4666	...	...	...	+·4664	+·0002
Licetus H	+·0383	+·0381	+ 2	+·0382	...	...
Stöfler *f*	+·0641	+·0633	+ 8	+·0637	...	...
Nicolai A	+·2966	...	...	...	+·2960	+ 6
Stöfler K	+·0567	+·0566	+ 1	+·0567	...	...
Orontius *d*	–·0829	–·0829	0	–·0829	...	...
Walter *a*	+·0093	+·0090	+ 3	+·0091	...	...
„ (sm. crater)‡	–·0025	–·0028	+ 3	–·0026	...	...
Hesiodus B	...	–·2680	...	...	–·2671	– 9
Purbach A	–·0295	–·0296	+ 1	–·0295	...	...
Nicollet	...	–·2003	...	...	–·1998	– 5
Thebit A	–·0789	–·0789	0	–·0789	–·0792	+ 3
Abenezra *b*	+·1638	+·1639	– 1	+·1638	+·1635	+ 3
Arzachael A	–·0247	–·0247	0	–·0247	...	...
Airy (sm. crater)§	+·0743	+·0746	– 3	+·0745	...	...
„ A	+·1277	+·1279	– 2	+·1278	+·1276	+ 2

* The most westerly of three confluent craters of which the most easterly is *b*.

† A small round crater west of *f* and *d*.

‡ The isolated crater on the south-east part of the floor.

§ To the east of *Airy*, almost in contact with the wall.

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1901MNRAS..62...41S

site illuminations tends to eliminate any systematic error depending upon either of these conditions. It is perhaps open to question whether the same amount of labour spent upon measuring and reducing several different plates independently, and taking the means of the resulting places, would not have given a closer approximation to the true values of the coordinates; but the greater part of the work was performed in the endeavour to obtain absolute places from a comparison of two photographs only, without appealing to telescopic measures at all. It was only when this failed that the method finally adopted suggested itself, and I think that the labour spent upon the reductions from that period has yielded an adequate return.

In the following table I give a comparison of the places finally obtained from the two plates with each other, and with those obtained at the telescope. The suffixes as before refer to the Plates II., III. respectively. The column headed "Authority" refers to the telescopic measures, and the initial B. denotes Professor Barnard, F. Dr. Franz, and S. myself.

Heliometer Measures by Dr. Franz, and Filar Micrometer Measures by by S. A. Saunder.

Values of η.						
Plate II.	Plate III.	II.–III.	Mean of II. & III.	Telescopic.	Photographic – Telescopic.	Authority.
−·7960	−·7964	+·0004	−·7962	...	...	
−·7285	−·7287	+ 2	−·7286	...	...	
−·7197	...	...	...	−·7202	+·0005	F.
−·7182	−·7182	0	−·7182	...	...	...
−·6774	−·6773	− 1	−·6773	...	...	...
−·6742	...	...	...	−·6749	+ 7	F.
−·6352	−·6352	0	−·6352	...	...	...
−·6345	−·6345	0	−·6345	...	...	...
−·5935	−·5935	0	−·5935	...	...	...
−·5546	−·5545	− 1	−·5545	...	...	...
...	−·4556	...	...	−·4555	− 1	S.
−·4405	−·4406	+ 1	−·4405	...	...	...
...	−·3725	...	...	−·3730	+ 5	S.
−·3677	−·3673	− 4	−·3675	−·3674	− 1	S.
−·3550	−·3548	− 2	−·3549	−·3552	+ 3	S.
−·3903	−·3903	0	−·3903	...	...	...
−·3025	−·3024	− 1	−·3025	...	...	...
−·2926	−·2926	0	−·2926	−·2929	+ 3	S.

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1901MNRAS..62...41S

	Values of ξ.					
	Plate II.	Plate III.	II.–III.	Mean of II. & III.	Telescopic.	Photographic —Telescopic.
Gassendi *z*	...	−·6522	...	...	−·6525	+ 3
Alpetragius B	−·1152	−·1156	+ 4	−·1154	...	...
Lubiniezky B	...	−·3864	...	...	−·3863	− 1
Dollond	+·2456	...	...	...	+·2458	− 2
Ptolemæus A	−·0139	−·0140	+ 1	−·0140	−·0142	+ 2
"	"	"	"	"	−·0139	− 1
Halley	+·0999	+·0994	+ 5	+·0996	...	...
Hipparchus C	+·1422	+·1426	− 4	+·1424	+·1423	+ 1
Euclides	...	−·4892	...	...	−·4886	− 6
Hipparchus L	+·1560	+·1565	− 5	+·1563	...	...
" G	+·1288	+·1292	− 4	+·1290	+·1287	+ 3
" *n*	+·0873	+·0871	+ 2	+·0872	...	...
Mösting A	−·0904	−·0904	0	−·0904	−·0900	− 4
Hipparchus H	+·0526	+·0528	− 2	+·0527	...	...
Gambart A	...	−·3209	...	...	−·3210	+ 1
Triesnecker B	+·0068	+·0067	+ 1	+·0067	+·0072	− 5
"	"	"	"	"	+·0073	− 6
Cayley	+·2602	...	...	...	+·2603	− 1
Murchison A	+·0199	+·0199	0	+·0199	...	...
Hortensius	...	−·4660	...	...	−·4662	+ 2
Sosigenes *a*	+·3144	...	...	...	+·3145	− 1
Bode B	−·0531	−·0528	− 3	−·0529	−·0529	0
" A	−·0201	−·0200	− 1	−·0201	−·0199	− 2
Proclus	+·7022	...	...	...	+·7022	0
Macrobius *a*	+·6107	...	...	...	+·6103	+ 4
Conon	+·0324	+·0326	− 2	+·0325	...	...
Bessel	+·2865	...	...	...	+·2856	+ 9
Aratus	+·0727	+·0728	− 1	+·0728	...	...
Archimedes A	−·0983	−·0983	0	−·0983	...	...
" *c*	−·0221	−·0223	+ 2	−·0222	...	...
" *b*	−·0268	−·0276	+ 8	−·0272	...	...
Kirch	−·0754	−·0755	+ 1	−·0754	...	...
Cassini *b*	+·0521	+·0520	+ 1	+·0521	...	...
" A	+·0635	+·0635	0	+·0635	...	...
" (small crater)*	+·0495	+·0492	+ 3	+·0494	...	...
Piazzi Smyth	−·0416	−·0418	+ 2	−·0417	...	...
Sharp A	...	−·4560	...	...	−·4566	+ 6

* Just outside Cassini to the north-east.

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Values of η.

Plate II.	Plate III.	II.–III.	Mean of II. & III.	Telescopic.	Photographic – Telescopic.	Authority.
...	−·2829	...	...	−·2833	+ 4	F.
−·2604	−·2605	+ 1	−·2605	...	...	...
...	−·2516	...	...	−·2515	− 1	S.
−·1817	...	...	...	−·1817	0	S.
−·1480	−·1480	0	−·1480	−·1479	− 1	B.
,,	,,	,,	,,	−·1478	− 2	S.
−·1390	−·1394	+ 4	−·1392	...	...	...
−·1285	−·1285	0	−·1285	−·1285	0	S.
...	−·1279	...	...	−·1283	+ 4	S.
−·1190	−·1192	+ 2	−·1191	...	...	...
−·0872	−·0872	0	−·0872	−·0875	+ 3	S.
−·0843	−·0841	− 2	−·0842	...	...	...
−·0558	−·0556	− 2	−·0557	−·0556	− 1	F.
−·0386	−·0382	− 4	−·0384	...	...	...
...	+·0167	...	...	+·0170	− 3	S.
+·0205	+·0204	+ 1	+·0204	+·0205	− 1	B.
,,	,,	,,	,,	+·0206	− 2	S.
+·0690	...	...	...	+·0690	0	S.
+·0697	+·0697	0	+0·697	...	...	...
...	+·1130	...	...	+·1128	+ 2	S.
+·1353	...	...	...	+·1349	+ 4	S.
+·1523	+·1520	+ 3	+·1521	+·1525	− 4	S.
+·1564	+·1568	− 4	+·1566	+·1569	− 3	S.
+·2772	...	...	...	+·2770	+ 2	F.
+·3342	...	...	...	+·3345	− 3	F.
+·3697	+·3693	+ 4	+·3695	...	...	...
+·3701	...	...	...	+·3704	− 3	S.
+·4004	+·4003	+ 1	+·4003	...	...	...
+·4699	+·4699	0	+·4699	...	...	...
+·5238	+·5237	+ 1	+·5238	...	...	...
+·5702	+·5697	− 5	+·5699	...	...	...
+·6321	+·6321	0	+·6321	...	...	...
+·6422	+·6422	0	+·6422	...	...	...
+·6488	+·6489	− 1	+·6488	...	...	...
+·6601	+·6603	− 2	+·6602	...	...	...
+·6671	+·6668	+ 3	+·6670	...	...	...
...	+·7380	...	...	+·7376	+	F.

1901MNRAS..62...41S

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1901MNRAS..62...41S

§ 7. *Discussion of Results obtained.*

It is satisfactory to note that no two positions of the same point are separated by as much as a second of arc, though in two or three cases this limit is approached. Some part of the discordances found may be real, depending, as has been shown by Dr. Franz, upon the elevation above or depression below the surface of the sphere most nearly representing the figure of the Moon. But it must be borne in mind that concordance between the places derived from the two photographs does not prove the accuracy of those places, otherwise the attempt to determine absolute places by securing such concordance would have succeeded. The real test is to be found in the agreement of places derived from the photographs with those obtained at the telescope.

The mean distance between these positions for the six points measured by Dr. Franz is $0''\cdot53$, these points being shown on only one plate each. For the nineteen points measured by myself there are twenty-eight photographic measures on the two plates, and the mean of the twenty-eight divergences is $0''\cdot38$, whilst for the ten points which have been measured on both plates the mean divergence between the telescopic and the mean photographic places is $0''\cdot32$. The mean divergence for the eighteen points measured on Plate II. is $0''\cdot37$; for the eighteen points measured on Plate III. it is $0''\cdot42$.

The greater divergence exhibited by the six points measured by Dr. Franz is, I think, chiefly due to the fact that they are not well defined on the photographs. There is not one of them that I should have selected as a really satisfactory point for accurate measurement on these particular plates, and I am pleased to find that the divergence is no greater. This I think may be taken as a satisfactory indication that the scale and orientation are well determined, so that divergences do not sensibly increase as we recede from the centre. I think also it may be fairly concluded from the whole comparison that the position of a point on any part of the surface may be determined from a single photograph on which it is well defined with an error less than one second of arc.

In estimating the advance thus made towards the accurate determination of selenographic positions it must be remembered that we are not dealing with stellar points; that in the most favourable cases, such as those here considered, we have to estimate the centre of a formation seldom less than 5″ or 6″ in diameter, and frequently more, subject to all the confusion produced by a transition from dazzling brightness to the blackest shade, and that therefore it is not to be expected that the position of a lunar formation should be capable of as accurate determination as that of a fixed star. I may perhaps be pardoned if I again call attention to the fact that the mean

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1901MNRAS..62...41S

distance between two positions of the same point found by Mädler and Lohrmann respectively, as determined from the eleven points measured as "points of the first order" not less than five times by each, was $5''\cdot2$, or almost exactly ten times the average discordance between the six points measured by Dr. Franz and my own determination of their positions, each from one photograph only, under conditions which were certainly not too favourable to the photographs.

The accuracy now attained is, I think, sufficient for such purposes as the preparation of large-scale maps, the computation of the Sun's altitude as required for determining the heights of mountains, and for the great problem to which these are subsidiary, the detection of possible changes on the surface.

But of greater importance than the increase of accuracy in the determination of a few isolated points is the enormous increase in the number of points which can now be accurately measured. With the old methods the computations required for the determination of "points of the first order" were so tedious that rougher methods were devised for determining points of the "second" and "third order," as they were called. These carried with them all the errors of the points of the first order from which they were measured, whilst introducing others due to the methods adopted for the reduction.

Beer and Mädler's map was based on 105 points of the first order, Schmidt's on 157. As a foundation for the outline maps which are being issued by the British Astronomical Association, I have already measured seventy-four points on an area equal to ·014 of the projection of the visible hemisphere (*B.A.A. Memoirs*, vol. x., 5th Report of the Lunar Section). This area is the corner of the S.W. quadrant near the centre of the disc, and includes the whole of *Hipparchus* and portions of the neighbouring formations. It was selected as a suitable region for the commencement of a scheme for remapping the whole surface, and if this is well taken up it is proposed to gradually extend the area. More points might have been measured on either of the plates, but I preferred to limit myself to those which could be measured with a fair approach to accuracy on both, and I have every reason to believe that these seventy-four points are determined with errors very little in excess of those in the list now given. With more photographs it will be possible to measure other points in the same area, as well as to increase the accuracy of those already measured. Some regions will perhaps give an even closer aggregation of measurable points, whilst the large seas will probably not give so many. But the density of distribution on the area already measured corresponds to more than 5,000 on the whole hemisphere. These will again serve as points of reference by which to fix the still more numerous points not at present shown in the photographs, but easily seen in a small telescope, and in which we are most likely to detect evidences of those changes which have been

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1901MNRAS..62...41S

so strongly suspected by the most trustworthy selenographers, and so ruthlessly rejected by the majority of the writers of astronomical text-books.

§ 8. *Focal Length of the Paris Telescope and Scale of the Photographs.*

In § 13 of my previous paper I gave the values found for the focal length of the photographic object glass of the Paris great Equatorial Coudé from the measures then made. These were vitiated by an erroneous assumption as to the size of the réseau square employed, which was really 5·040 mm., instead of 5·000 mm., as well as by the errors attendant upon limb measures. With the scale values now adopted it appears that a side of the réseau square on Plate II. corresponds to $57''\cdot55$, and on Plate III. to $57''\cdot59$. The values which I obtain for the focal length of the telescope are 18·064 m. on 1899 February 18, and 18·052 m. on 1895 September 9. The temperatures on these two nights were probably very different, but the difference of focal length is of the opposite sign to that which would be expected from this cause alone.

§ 9. *Conclusion.*

The computations have been almost entirely performed with a "Brunsviga," which is admirably adapted, not only for computing expressions of the form $Ax+By+Cz$, of which a large part of the work has consisted, but also for forming the normal equations in a least square solution, and for the eliminations according to Gauss' method. The computations have been carried to not less than five places throughout; where there appeared to be special danger of cumulative error more figures have been retained. The values of ξ, η given by the photographic plates have been doubly computed—first by applying the differential corrections to the values computed with the approximate plate constants, and secondly directly from the measured coordinates with the corrected values of the plate constants.

I should hope that the positions now obtained are sufficient both in number and in accuracy to allow of the determination of the constants of a plate by the method proposed by Professor Turner, and described in § 9 of my previous paper, by which a considerable reduction is effected in the amount of computation required. My next step will be to endeavour to reduce a plate in this way. If I am successful progress in the future will be more rapid than it has been in the past.

The plates, as stated in my previous paper, are positive copies on glass, kindly made for me by Mr. Bellamy, of the University Observatory, Oxford, from Paris negatives, for which I am indebted to the generosity of M. Loewy.

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1901MNRAS..62...41S

The photographic measures were made with an astrographic micrometer of the pattern in use at Greenwich and Oxford, the property of the University Observatory, Oxford, and kindly lent me by Professor Turner, who has not only placed the resources of the Observatory at my disposal, but has supplemented them with a large share of personal advice and assistance, for all of which my most hearty thanks are due.

§ 10. *Comparison of Results with those in Breslau "Mitteilungen," Vol. I.*

Since this paper was written copies have been received in England of the first volume of the *Mitteilungen* of the Breslau Observatory, in which Dr. Franz gives the places of 150 lunar formations as determined by the measurement of five Lick photographs, and it has been with great interest that I have compared his methods and results with my own. In what may be considered the essentials of the methods we agree: we have both made our work independent of direct measures of the limb, and have determined the constants of our plates by comparing the photographic measures of a number of points with the positions of the same points determined by telescopic measures from *Mösting A*; but in the details there are many points of difference.

Professor Franz's telescopic measures have been made with the heliometer, my own with the filar micrometer, the bisection of two formations with the latter instrument being an entirely different operation from the superposition of their images with the former. Dr. Franz has been able to measure eight points near the limb. I have been obliged to measure a greater number nearer the centre, although I have been able to avail myself to a limited extent of his published results, as explained above.

The photographs which Dr. Franz has measured have all been taken near the time of full Moon, so that a great number of the objects measured have appeared only as light spots, no detail being visible. My photographs show but little more than half the disc illuminated, and I have not measured a crater unless a considerable portion of the wall could be made out.

In measuring the photographs Dr. Franz has bisected the craters with a thread moved by a screw, the reading of which, added to that of an accurate scale on which the microscope was carried, gave the measured coordinate. I have measured with a réseau and a graduated cross in the eyepiece, the setting being made either by getting equal readings on opposite arms where they cut the crater walls, or by setting the crater symmetrically in one of the squares formed by the longer (fifth or tenth) division lines.

The reductions have also been performed by quite independent methods.

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1901MNRAS..62...41S

It would thus seem unlikely that the two sets of places should be affected by the *same* systematic errors.

In order to compare the results it was necessary for me to give relative weights to my own photographic and telescopic measures. I find that the probable error of a coordinate measured on one photographic plate is 0″·09, as determined from the divergences between the two plates. The corresponding probable error for the mean of four telescopic measures was found in § 2 to be 0″·10. But it has already been pointed out that the measure of the discordance between the two plates may give too high an estimate of the value of the absolute results obtained, since errors in the constants give systematic errors in the results, and the method of solution renders it probable that these will be in the same direction in the two plates. I have therefore given to a coordinate obtained from one plate half the weight of the mean of four telescopic measures. I may note in passing that Dr. Franz gives no weight to his photographic measures as compared with those made with the heliometer.

In addition to *Mösting A* and the six points already referred to, there are seventeen points that Dr. Franz and I have both measured. In the following table I compare the positions which result from this combination of my own measures of these points with those found by Dr. Franz :—

	Values of ξ.			Values of η.		
	J. F.	S. A. S.	F.-S.	J. F.	S. A. S.	F.-S.
Nicollet	−·1997	−·2000	+·0003	−·3732	−·3728	−·0004
Thebit A	−·0794	−·0791	− 3	−·3677	−·3674	− 3
Airy (sm. crater)	+·0746	+·0745	+ 1	−·3028	−·3025	− 3
Airy A	+·1278	+·1277	+ 1	−·2930	−·2928	− 2
Lubiniezky B	−·3859	−·3863	+ 4	−·2520	−·2515	− 5
Ptolemæus A	−·0138	−·0140	+ 2	−·1480	−·1479	− 1
Hipparchus C	+·1422	+·1423	− 1	−·1288	−·1285	− 3
Euclides	−·4880	−·4888	+ 8	−·1287	−·1282	− 5
Hipparchus G	+·1289	+·1288	+ 1	−·0874	−·0874	0
Gambart A	−·3211	−·3210	− 1	+·0167	+·0169	− 2
Cayley	+·2599	+·2603	− 4	+·0688	+·0690	− 2
Sosigenes *a*	+·3139	+·3145	− 6	+·1351	+·1350	+ 1
Bode B	−·0530	−·0529	− 1	+·1520	+·1523	− 3
„ A	−·0198	−·0200	+ 2	+·1562	+·1568	− 6
Bessel	+·2855	+·2859	− 4	+·3699	+·3703	− 4
Aratus	+·0724	+·0728	− 4	+·4005	+·4003	+ 2
Archimedes A	−·0984	−·0983	− 1	+·4698	+·4699	− 1

In no case does the discordance between two positions amount to 1″ of arc (geocentric). The only formation for which this value is approached is *Euclides*, which was not well defined on

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1901MNRAS..62...41S

Dr. Franz's photographs. This one may conjecture to have been due to the brightness of its surroundings.

The comparison reveals a systematic difference of ·0002 in the ordinates, but is consistent with the supposition that the differences in the abscissæ are accidental.

This difference in the ordinates is slightly increased when the comparison is made between Dr. Franz's positions and my telescopic positions only, whilst it would have practically disappeared from the photographic measures had I adopted the ordinate of the centre derived in § 5 from the photographic measures of the six points measured with the heliometer by Dr. Franz.

The result therefore justifies my rejection of these photographic measures so far as the abscissa of the centre is concerned, but gives reason for suspecting a real systematic difference of about $0''\cdot2$ in the ordinates between Dr. Franz's positions found with the heliometer and mine found with the filar micrometer.

The mean geocentric distance between the positions of the same point, as found by Dr. Franz and myself, is $0''\cdot39$, or less than one-thirteenth of the corresponding mean distance between positions found by Lohrmann and Mädler. There was also a systematic difference of a different kind between the ordinates found by these observers about fifteen times as great as that between Dr. Franz and myself, Lohrmann placing his points further from the equator than Mädler.

That small systematic differences should appear is not surprising. Dr. Franz calls attention to their existence between different parts of his own work, and I have already noted the same feature with regard to my work in the *Fifth Report of the Lunar Section of the British Astronomical Association.*

It says much for the general accuracy of the method that both these and the accidental differences should be as small as they are; and it is a great gratification to me to find my own work in such close agreement with that of one who has, I believe, given more attention to selenographic measurements than any other living astronomer, and to whom we owe the best determinations both of the physical libration and of the figure of the Moon.

Recent Observations of the Position of Nova Aurigæ with the 40-inch Telescope of the Yerkes Observatory. By E. E. Barnard.

Shortly after the announcement of Dr. Anderson's discovery of *Nova Aurigæ* in 1892 February Mr. Burnham made a careful set of measures of its position with reference to some thirteen stars near it.

The results of this work are given by him in a paper in

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